Hunters Hill High School

Student Number

2022 TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I – 10 marks (pages 3–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8-36)

- Attempt Questions 11–31
- Allow about 2 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 A geometric sequence has a common ratio of -1.5 and the second term is 30. Find the fifth term.

(A) 67.5

(B) -101.25

(C) -67.5

(D) 151.875

2 Which of the following expressions correctly represents the derivative of 3^{2x}?

(A) $\ln 9 \cdot 9^x$

(B) $\ln 6 \cdot 3^{2x}$

(C) $2 \cdot 3^{2x}$

(D) 6^{2x}

3 The function $f(x) = \ln x$ is transformed to $g(x) = \ln(x + 3) - 4$ by a horizontal translation of 3 units, followed by a vertical translation of 4 units.

Which of the following correctly shows the directions of the translations?

	Horizontal translation of 3 units	*Vertical translation of 4 units*
(A)	Right	Up
(B)	Right	Down
(C)	Left	Up
(D)	Left	Down

4 Which interval gives the domain of the function $y = \dfrac{3}{\sqrt{x+3}}$

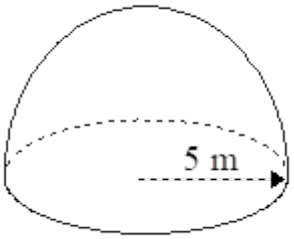
(A) $[-3, \infty)$

(B) $(-\infty, -3)$

(C) $(-3, \infty)$

(D) $(-\infty, -3]$

5 Calculate the volume of a hemisphere with a radius of 5 m. Answer correct to one decimal place.



(A) 261.8 m^3

(B) 523.6 m^3

(C) 785.4 m^3

(D) 1048 m^3

6 A particle is moving with velocity $v = t^2 - 10t + 21$ for $t \geq 0$. For what value(s) of t is the particle stationary?

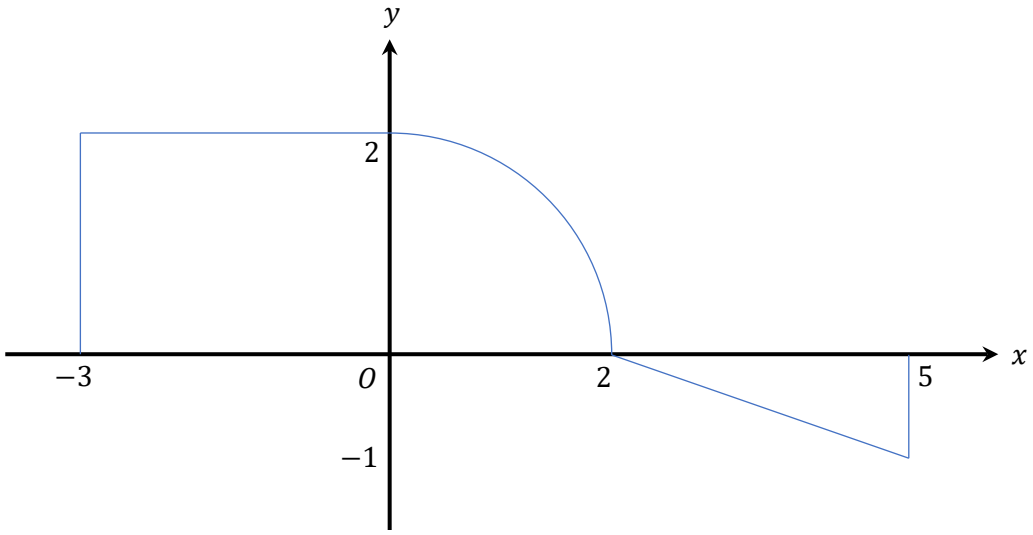
(A) When $t = 3$

(B) When $t = 5$

(C) When $t = 3$ and $t = 7$

(D) When $t = 5$ and $t = 7$

7 The diagram below shows the graph of $y = f(x)$. The graph of $y = f(x)$ consists of a sector, a rectangle, and a triangle.



What is the exact value of $\int_{-3}^{5} f(x)dx$?

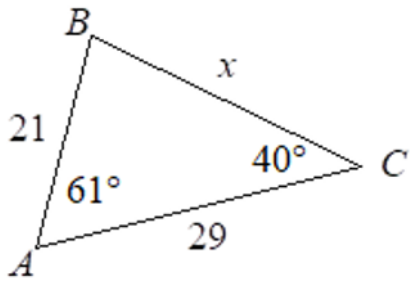
(A) $\pi + \frac{9}{2}$

(B) $\pi + \frac{15}{2}$

(C) $\pi - \frac{15}{2}$

(D) $\pi - \frac{9}{2}$

8 What is the correct expression for the value of x in ΔABC?



(A) $x = \dfrac{29 \sin 79°}{\sin 61°}$

(B) $x = \dfrac{21 \sin 40°}{\sin 61°}$

(C) $x = \dfrac{29 \sin 61°}{\sin 79°}$

(D) $x = \dfrac{21 \sin 40°}{\sin 79°}$

9 Let $f(x) = x^3 - (k^2 - 4)x + 1$.

For which of the values of k below is $f(x)$ many-to-one?

(A) $k = 2$

(B) $k = -2$

(C) $k = 1$

(D) $k = -4$

10 For $f(x)$, a differentiable function, $f'(x) < 0$ and $f''(x) > 0$.

If a and b are in the domain of $f(x)$ and $a < b$, which statement is NOT always true?

(A) $f(a) > f(b)$

(B) $f'(b) < f'(a) < 0$

(C) $f'(a) < \dfrac{f(b) - f(a)}{b - a}$

(D) $f'(b) > \dfrac{f(b) - f(a)}{b - a}$

End of Section I

Trial Examination

2022

Student Number

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11–31
Allow about 2 hours and 45 minutes for this section

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Solve $x + \dfrac{10 + x}{3} = 18$. **2**

Question 12 (2 marks)

Evaluate $\displaystyle\int_0^1 \frac{x^2}{1 + x^3}\,dx$ and express your answer in exact form. **2**

Question 13 (2 marks)

Find the equation of the tangent to $y = \sin 2x$ at the point $\left(\frac{\pi}{2}, 0\right)$. **2**

Question 14 (2 marks)

Prove that $\sec\theta - \tan\theta \sin\theta = \cos\theta$. **2**

Question 15 (3 marks)

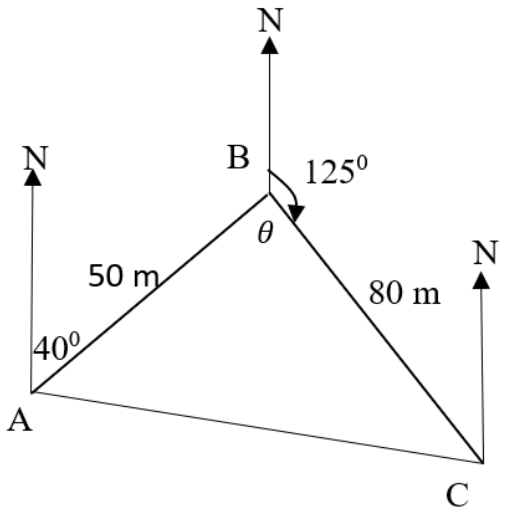
After completing a numeracy test, Hanna had achieved a z-score of -2.

(a) How does Hanna's results compare with those of her peers? Explain your answer in terms of the mean and standard deviation. **1**

(b) If the mean test mark was 86 and the standard deviation was 4, calculate Hanna's actual mark. **2**

Question 16 (6 marks)

The diagram below shows the plan for a triangular park. Fence posts have been located at points A, B and C.
AB is 50 m and BC is 80 m. The bearing of B from A is 040° and the bearing of C from B is 125°.



(a) What is the size of angle θ? **1**

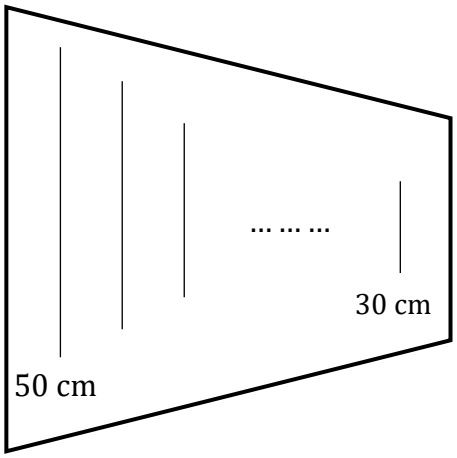
(b) What is the true bearing of point A from point B? **1**

(c) Find the area of the park. Correct your answer to one decimal place. **2**

(d) Find the length of AC. Correct your answer to one decimal place. **2**

Question 17 (4 marks)

A simple instrument has many strings, attached as shown in the diagram. The difference between the lengths of adjacent strings is a constant, so that the lengths of the strings are the terms of an arithmetic sequence.



The shortest string is 30 cm long and the longest string is 50 cm. The sum of the lengths of all the strings is 1240 cm.

(a) Show that there are 31 strings on the instrument. **2**

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(b) Find the difference in length between adjacent strings. **2**

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Question 18 (2 marks)

Calculate the limiting sum of the geometric series $4 - 2 + 1 - \frac{1}{2} + \frac{1}{4} - \cdots$ **2**

Question 19 (6 marks)

A discrete random variable X has the probability distribution table shown.

x	1	2	3	4
$P(X = x)$	0.1	p^2	0.54	$1 - 2p$

(a) Explain why $p^2 - 2p + 0.64 = 0$. Show mathematical working to support your answer. **2**

(b) Hence, show that $p = 0.4$ or $p = 1.6$ and determine which of these solutions is invalid, giving a reason. **2**

(c) If $p = 0.4$, calculate the expected value of X. **2**

Question 20 (2 marks)

Find $\int \sqrt{4x+1}\ dx$. **2**

Question 21 (3 marks)

The amount of money gained by an entrepreneur doubles every day. On day 1, she earns $9 and on day 2 she earns $18.

On which day did she first earn more than a total of $1 million?

3

Question 22 (6 marks)

The queuing time, X minutes, of a customer at a checkout in a store has the probability density function

$$f(x) = \begin{cases} kx(4-x) & \text{for } 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the value of k is $\frac{3}{32}$. **2**

(b) Find $P(1 < X < 2)$. **2**

(c) By finding the expected wait time, determine whether the customer is more likely to wait less than 1 minute, or more than 1 minute. **2**

Question 23 (7 marks)

Coal is extracted from a mine at a rate that is proportional to the amount of coal remaining in the mine. Hence, the amount R remaining after t years is given by

$$R = 250e^{-kt}$$

k is a positive constant and R is measured in tonnes.

(a) If 80% of the coal remains after 2 years, show that the value of k is approximately 0.1116. **2**

(b) Find the amount of coal remaining after 8 years, correct to the nearest tonne. **1**

(c) At what rate is the amount of coal decreasing after 1 year? Give your answer correct to the nearest whole number. **2**

(d) The coal mine is abandoned when the amount of coal remaining is less than 80 tonnes. After how many years will the coal mine be abandoned? Express your answer in years, correct to one decimal place. **2**

Question 24 (2 marks)

Jemma is a political advisor who studies the effects of time on television over a month on the approval rating of six politicians. The data is shown below.

Time (in minutes)	20	10	70	15	5	80
Approval rating (%)	25	50	55	30	80	85

(a) Using a calculator, find Pearson's correlation coefficient, r. Give your answer correct to 4 decimal places. **1**

(b) Jemma concludes that a new politician should appear on television as often as possible to gain a high approval rating. Do you agree? Give a reason. **1**

Question 25 (2 marks)

A curve $y = f(x)$ passes through $\left(\frac{\pi}{2}, -\frac{\pi}{2}\right)$ and has the gradient function $\frac{dy}{dx} = 4\cos 2x + 1$. Find the equation of the curve. **2**

Question 26 (9 marks)

Consider the function $f(x) = 4x^2 - 2x^3$.

(a) Show that the point of inflection is at $\left(\frac{2}{3}, \frac{32}{27}\right)$. **2**

(b) Sketch the graph of the curve $y = f(x)$, labelling the stationary points, the point of inflection, and the x-intercept. **4**

(c) Hence, or otherwise, state the domain for which the function is increasing. **1**

(d) Find the coordinates of the new point of inflection if $y = f(x)$ is transformed to $y = -f(-x)$. **2**

Question 27 (5 marks)

In a bag there are two fair coins, for which $P(T) = 0.5$.

There is also a third, fake coin where both faces have a tail.
So, for this coin $P(T) = 1$.

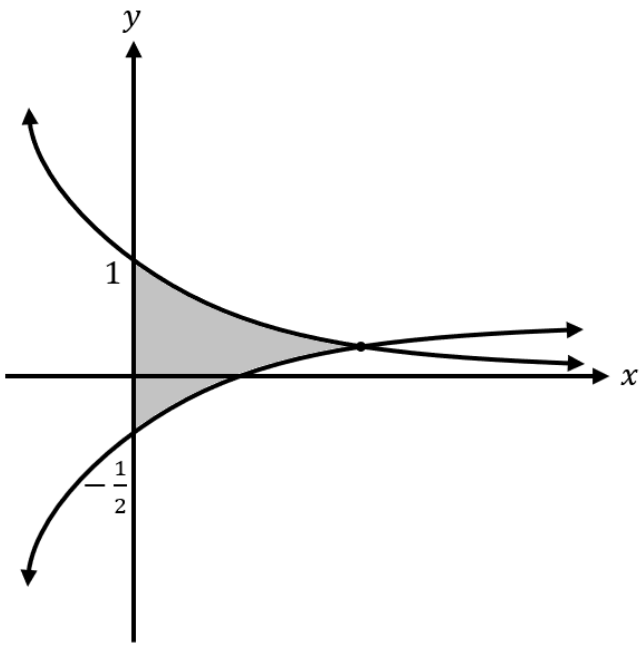
(a) A coin is selected a random and tossed. What is the probability of getting a tail? **2**

(b) A coin is selected at random and tossed. A tail comes up. **3**

What is the probability that the fake coin was chosen?

Question 28 (5 marks)

The diagram below shows the graphs of $y = 2^{-x}$ and $y = \frac{1}{2} - 2^{-x}$.



(a) By solving simultaneously, show that the point of intersection occurs when $x = 2$. **2**

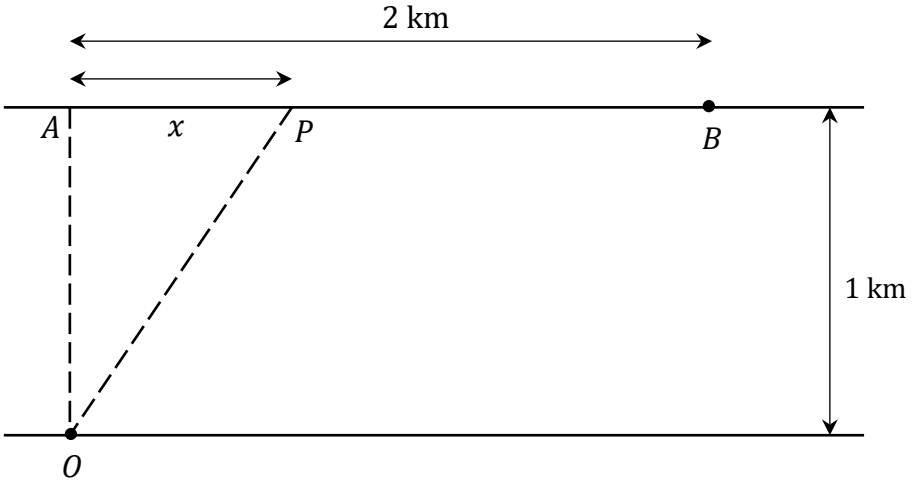
(b) Hence, find the area of the shaded region. **3**

Question 29 (7 marks)

The diagram shows a straight section of a river, one kilometre wide. Tamara is at point O on one bank and she wishes to reach point B on the opposite bank. The point A is directly opposite O and the distance from A to B is 2 kilometres.

Tamara can row at 6 km/h and jog at 10 km/h. Show intends to row in a straight line to a point P on the opposite bank, and then jog directly from P to B.

Let $AP = x$



(a) Show that the total distance Tamara will travel to reach point $B$is given by **2**

$$d = \sqrt{x^2 + 1} + (2 - x)$$

(b) Hence, explain why the time T, in hours, for Tamara to reach point B is **1**

$$T = \frac{\sqrt{x^2 + 1}}{6} + \frac{2 - x}{10}$$

(c) Find the value of x such that Tamara minimises the time taken to complete her journey and show that this value of x is a minimum. **3**

(d) Hence, determine the time taken to complete the journey in minimum time. **1**

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Question 30 (5 marks)

A company uses a machine to produce bottled soft drinks. The amount of liquid poured into each bottle by the machine is normally distributed, with a mean of 600 mL and a standard deviation of 5 mL.

The table below gives the probability that a random variable (that is normally distributed with mean 0 and standard deviation 1) is less than z for different values of z.

z	*first decimal place*									
	.0	**.1**	**.2**	**.3**	**.4**	**.5**	**.6**	**.7**	**.8**	**.9**
0.	0.500	0.540	0.579	0.618	0.655	0.692	0.726	0.758	0.788	0.816
1.	0.841	0.864	0.885	0.903	0.919	0.933	0.945	0.955	0.964	0.971
2.	0.977	0.982	0.986	0.989	0.992	0.994	0.995	0.997	0.997	0.998
3.	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000

(a) What percentage of bottles would have between 595 mL and 602 mL of liquid? **3**

(b) To comply with regulations, each bottle must contain at least x mL of liquid. The company is informed that 5.5% of bottles do not meet this regulation. **2**

Find the value of x.

Question 31 (8 marks)

A weight is attached to a spring that is connected to the ground. The weight is pushed down towards the ground, and then released so that it bounces up and down.

The spring stretches and contracts such that the distance of the weight from the ground, x (in metres), varies sinusoidally with time, t (in seconds).

It takes 2 seconds for the weight to go from its highest point of 9 centimetres to its lowest point of 5 centimetres.

The motion of the weight can be modelled by the equation

$$x = a\sin(bt + c) + d$$

(a) Find the values of a, b and d. **4**

(b) Find a possible value of c, given that the weight was initially pushed towards the ground before being released. **2**

(c) Hence, sketch the graph of the weight's motion against time, for $0 \leq t \leq 4$. **2**

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End of paper

12MAA – Trial HSC Examination, 2022
Solutions and Marking Guidelines

SECTION I

1.	B
2.	A
3.	D
4.	C
5.	A
6.	C
7.	A
8.	C
9.	D
10	B

SECTION II

11.	$x + \dfrac{10+x}{3} = 18$ $3x + 10 + x = 54$ $4x = 44$ $x = 11$	2 – Correct solution obtained with mathematical working. 1 – Correct step towards solution provided.
12.	$\displaystyle\int_0^1 \frac{x^2}{1+x^3}\,dx = \frac{1}{3}\int_0^1 \frac{3x^2}{1+x^3}\,dx$ $= \left[\dfrac{1}{3}\ln\lvert 1+x^3\rvert\right]_0^1$ $= \dfrac{1}{3}\ln 2$	2 – Correct mathematical working and solution provided. 1 – Correct mathematical working provided.
13.	$y = \sin 2x$ $y' = 2\cos 2x$ When $x = \frac{\pi}{2}, y' = -2$ ∴ Equation of tangent is: $y - 0 = -2\left(x - \dfrac{\pi}{2}\right)$ $y = -2x + \pi$	2 – Correct mathematical working and solution provided 1 – Correct derivative obtained.
14.	$LHS = \sec\theta - \tan\theta\sin\theta$ $= \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}\cdot\sin\theta$ $= \dfrac{1-\sin^2\theta}{\cos\theta}$ $= \dfrac{\cos^2\theta}{\cos\theta}$ $= \cos\theta$ $= RHS$	2 – Correct proof provided. 1 – Correct common denominator or equivalent obtained.

15.	**a)**	She scored 2 z-scores/σ **below the average**.	1 – Correctly provides mark as below average and by 2 σ.
	b)	Can use formula for z-score. $z = \frac{x-\mu}{\sigma}$ $-2 = \frac{x-86}{4}$ $x = 78$	2 – Correct solution determined with mathematical working. 1 – Correct solution determined only, or only correct mathematical working provided.
16.	**a)**	$\theta = 40° + 55° = 95°$	1 – Correct solution determined.
	b)	Bearing from B to $A = 125° + \theta$ $= 125° + 95°$ $= 220°$	1 – Correct solution determined.
	c)	$A = \frac{1}{2}ab\sin C$ $= \frac{1}{2} \cdot 50 \cdot 80 \cdot \sin 95°$ $= 1992.4 \text{ m}^2$	1 – Use of area of triangle formula provided. 2 – Correct mathematical working and solution obtained.
	d)	Using the cosine rule: $AC^2 = 50^2 + 80^2 - 2 \cdot 50 \cdot 80 \cdot \cos 95°$ $= 9597.2459 \ldots$ $AC = 98.0 \text{ m}$	2 – Correct equation with cosine rule and solution provided. 1 – Correct equation with cosine rule provided.
17.	**a)**	$S_n = \frac{n}{2}(a+l)$ $1240 = \frac{n}{2}(30+50)$ $1240 \cdot \frac{2}{80} = n$ $n = 31$	2 – Correct solution provided with mathematical working 1 – Use of sum formula for AP provided.
	b)	$T_n = a + (n-1) \cdot d$ $50 = 30 + (31-1) \cdot d$ $d = \frac{2}{3} \text{ cm}$ Note: $d = -\frac{2}{3}$ also accepted if the terms are reversed	2 – Correct solution and mathematical working provided. 1 – Correct mathematical working provided.
18.		$S_\infty = \frac{4}{1-\left(-\frac{1}{2}\right)}$ $= \frac{4}{\left(\frac{3}{2}\right)}$ $= \frac{8}{3}$	2 – Correct solution and mathematical working provided. 1 – Correct mathematical working provided.
19.	**a)**	Sum of all probability in a discrete probability distribution = 1. $0.1 + p^2 + 0.54 + 1 - 2p = 1$ $0.64 + p^2 - 2p = 0$	2 – Correct reasoning with mathematical working provided. 1 – Either one of the above provided.

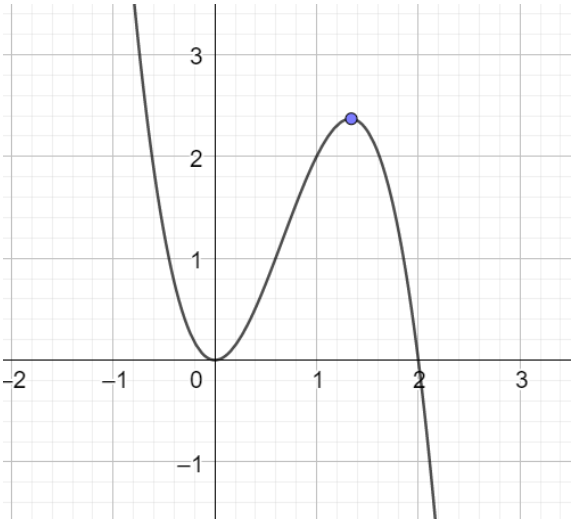
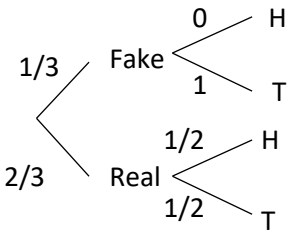
b)	$p^2 - 2p + 0.64 = 0$ $p = \frac{2 \pm \sqrt{4 - 4(0.64)}}{2}$ $= \frac{2 \pm 1.2}{2}$ $= 1.6, 0.4$ The probability of each possible score most be between 0 and 1. If $p = 1.6$, then $P(X = 2) > 0$ and $P(4) < 0$ which is not possible. Note: No mark awarded for reasoning if it is stated that p must be between 0 and 1. This is because p is NOT the probability.	2 – Correct solution provided with correct reasoning. 1 – Correct application of quadratic formula provided.
c)	$E(X) = 1 \cdot 0.1 + 2 \cdot 0.4^2 + 3 \cdot 0.54 + 4 \cdot (1 - 2 \cdot 0.4)$ $= 2.84$	2 – Correct solution and mathematical working provided. 1 – Correct mathematical working provided.
20.	$\int \sqrt{4x+1}\ dx = \int (4x+1)^{\frac{1}{2}}\ dx$ $= \frac{1}{4}\int 4(4x+1)^{\frac{1}{2}}\, dx$ $= \frac{1}{4}\frac{(4x+1)^{\frac{3}{2}}}{\frac{3}{2}} + C$ $= \frac{1}{6}(4x+1)^{\frac{3}{2}} + C$	2 – Correct solution and mathematical working provided. 1 – Correct mathematical working provided (coefficient or substitution) or integral obtained without ¼.
21.	$9 + 18 + 36 + 72 + \cdots$ $S_n = \frac{a(1-r^n)}{1-r}$ $1\,000\,000 < \frac{9(1-2^n)}{1-2}$ $-\frac{1000000}{9} > 1 - 2^n$ $-1 - \frac{1000000}{9} > -2^n$ $2^n > 1 + \frac{1000000}{9}$ $n > \frac{\ln 111112.11}{\ln 2}$ $n > 16.76$ ∴ First earns \$1 million after 17 days.	3 – Correct solution obtained with mathematical working. 2 – Error made, or correctly calculated the value for n with incorrect interpretation of question (i.e.: finding the term instead of using the total) 1 – Correct application of formula evident.
22. **a)**	$\int_0^4 kx(4-x)dx = 1$ $\left[2kx^2 - \frac{kx^3}{3}\right]_0^4 = 1$	2 – Correct solution and mathematical working provided. 1 – Correctly integrate expression or recognises the area between 0 and 4 is 1.

	$32k - \frac{64}{3}k = 1$ $\frac{32}{3}k = 1$ $k = \frac{3}{32}$	0 – Answer provided with no or incorrect mathematical working
b)	$P(1 < X < 2) = \int_1^2 \frac{3}{32}x(4-x)dx$ $= \left[\frac{3}{16}x^2 - \frac{1}{32}x^3\right]_1^2$ $= \frac{2}{16}(2)^2 - \frac{1}{32}(2)^3 - \left(\frac{3}{16} - \frac{1}{32}\right)$ $= 0.34375$	2 – Correct solution obtained with mathematical working. 1 – Correctly integrates expression. Or, if $P(X < 1)$ is found and correct reasoning provided.
c)	$E(X) = \int_0^4 \frac{3}{32}x^2(4-x)dx$ $= \frac{3}{32}\left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_0^4$ $= \frac{3}{32}\left(\frac{256}{3} - \frac{256}{4}\right)$ $= 2$ Average wait time is 2 minutes, so more likely to wait more than 1 minute.	2 – Correctly determines expected value and provides reasoning. 1 – Correct integral for expected value provided.
23. a)	When $t = 2, R = 200$ $200 = 250e^{-2k}$ $0.8 = e^{-2k}$ $\ln 0.8 = -2k$ $k = \frac{\ln 0.8}{-2} = 0.11157 \approx 0.1116$	2 – Correct solution obtained with mathematical working. 1 - Correctly uses given information for equation.
b)	When $t = 8, R = 250e^{-0.1116 \cdot 8} = 102$ (nearest tonne)	1 – Correct solution obtained.
c)	$\frac{dR}{dt} = -250ke^{-kt}$ When $t = 1$, $\frac{dR}{dt} = -250 \cdot 0.1116 \cdot e^{-0.1116 \cdot 1}$ $= -25$ tonnes/year	2 – Correct derivative and solution obtained. 1 – Correct derivative or solution obtained.
d)	When $R < 80$ $80 > 250e^{-kt}$ $\frac{80}{250} > e^{-0.1116 \cdot t}$ $\ln\frac{8}{25} > -0.1116t$ $t > 10.2$	2 – Obtains equation/inequality and solves for t 1 – Correct mathematical working provided.

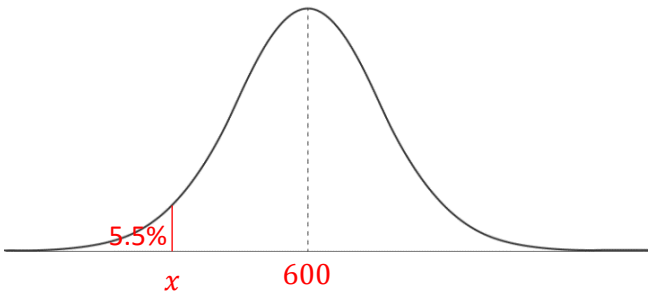
<table>
<tr><td></td><td>∴ After 10.2 years the coal mine will have less than 80 tonnes of coal.</td><td></td></tr>
<tr><td>24. a)</td><td>0.4099</td><td>1 – Correct r obtained.</td></tr>
<tr><td>b)</td><td>No because the amount of time on TV does not strongly correlate to a good approval rating as there is no strong correlation.</td><td>1 – Correct reasoning and choice.</td></tr>
<tr><td>25.</td><td>$$\frac{dy}{dx} = 4\cos 2x + 1$$
$$y = 2\sin 2x + x + C$$
When $x = \frac{\pi}{2}, y = -\frac{\pi}{2}$
$$-\frac{\pi}{2} = 2\sin\pi + \frac{\pi}{2} + C$$
$$C = -\pi$$
$$\therefore y = 2\sin 2x + x - \pi$$</td><td>2 – Correct value of C obtained with correct equation provided.

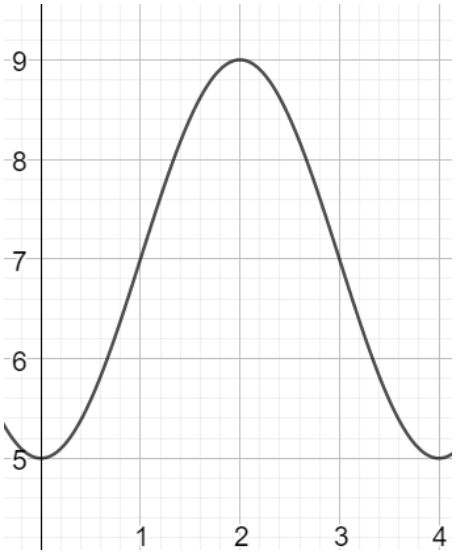
1 – Correct integral obtained.</td></tr>
<tr><td>26. a)</td><td>$$f(x) = 4x^2 - 2x^3$$
$$f'(x) = 8x - 6x^2$$
$$f''(x) = 8 - 12x$$
For points of inflection, let $f''(x) = 0$
$$0 = 8 - 12x$$
$$x = \frac{2}{3}$$
When $x = \frac{2}{3}, f\left(\frac{2}{3}\right) = \frac{32}{27}$
<table><tr><td>x</td><td>$\frac{2^-}{3}$</td><td>$\frac{2}{3}$</td><td>$\frac{2^+}{3}$</td></tr><tr><td>$f''(x)$</td><td>+</td><td>0</td><td>−</td></tr></table>∴ as concavity changes at $x = \frac{2}{3}$, there is a point of inflection at $\left(\frac{2}{3}, \frac{32}{27}\right)$</td><td>2 – Correct mathematical working provided to determine point of inflection.

1 – Correct first and second derivative obtained.</td></tr>
</table>

b)	 For stationary points, let $f'(x) = 0$ $$8x - 6x^2 = 0$$ $$x(4 - 3x) = 0$$ $$x = 0, \frac{4}{3}$$ When $x = 0, f''(0) = 8 > 0$ (concave up) When $x = \frac{4}{3}, f''\left(\frac{4}{3}\right) = -8 < 0$ (concave down) $\therefore$ Minimum at $(0,0)$ and maximum at $\left(\frac{4}{3}, \frac{64}{27}\right)$ x-intercept at $x = 4$	2 – Correct stationary points shown. 1 – Correct point of inflection shown. 1 – Correct x-intercept shown.
c)	Function is increasing in the interval $\left(0, \frac{4}{3}\right)$	1 –Correct domain provided.
d)	Reflection about x and y axis. $$\left(-\frac{2}{3}, -\frac{32}{27}\right)$$	2 – Correct mathematical working with correct point. 1 – Correct mathematical working provided or x or y ordinate provided.
27. a)	 $$P(T) = \frac{1}{3} + \frac{2}{3} \cdot \frac{1}{2} = \frac{2}{3}$$	2 – Correct solution obtained with mathematical working or tree diagram. 1 – Correct mathematical working towards solution provided.
b)	$$P(\text{Fake}\|T) = \frac{P(\text{Fake \& Tail})}{P(\text{Tail})}$$ $$= \frac{\left(\frac{1}{3}\right)}{\left(\frac{2}{3}\right)}$$ $$= \frac{1}{2}$$	3 – Correct solution provided with correct mathematical working. 2 – Correct application of conditional probability or equivalent. 1 – Evidence of conditional probability provided.

28. **a)**	$2^{-x} = \frac{1}{2} - 2^{-x}$ $2 \cdot 2^{-x} = \frac{1}{2}$ $2^{-x} = \frac{1}{4}$ $x = 2$ Using logarithms or recognising $2^{-2} = \frac{1}{4}$	2 – Correct mathematical working and solution obtained 1 – Equates equations or equivalent to solve simultaneously.
b)	$A = \int_0^2 2^{-x} - \left(\frac{1}{2} - 2^{-x}\right) dx$ $= \int_0^2 2 \cdot 2^{-x} - \frac{1}{2} dx$ $= \int_0^2 2^{1-x} - \frac{1}{2} dx$ $= \left[-\frac{2^{1-x}}{\ln 2} - \frac{1}{2}x\right]_0^2$ $= -\frac{1}{2\ln 2} - 1 - \left(-\frac{2}{\ln 2}\right)$ $= \frac{3}{2\ln 2} - 1$	3 – Correct mathematical working and solution provided. 2 – Any correct integration for relevant area provided. 1 – Any correct expression for relevant area provided.
29. **a)**	$PB = 2 - x$ $OP^2 = AP^2 + OA^2$ $OP = \sqrt{x^2 + 1}$ Total distance is: $d = PB + OP$ $= \sqrt{x^2 + 1} + 2 - x$	2 – Correct expression for distance determined. 1 – One expression derived and provided.
b)	$Time = \frac{Dist.}{Speed}$ $T_{OP} = \frac{\sqrt{x^2 + 1}}{6}$ and $T_{PB} = \frac{2 - x}{10}$	1 – Correct reasoning provided.
c)	$\frac{dT}{dx} = \frac{1}{6} \cdot \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}} \cdot 2x - \frac{1}{10}$ $= \frac{x}{6\sqrt{x^2 + 1}} - \frac{1}{10}$ For minimum time, let $\frac{dT}{dx} = 0$ $0 = \frac{x}{6\sqrt{x^2 + 1}} - \frac{1}{10}$ $\frac{6}{10} = \frac{x}{\sqrt{x^2 + 1}}$ $\frac{36}{100} = \frac{x^2}{x^2 + 1}$ $36x^2 + 36 = 100x^2$ $64x^2 = 36$	3 – Shows that x gives minimum time 2 – Differentiates both expressions and correctly finds the value of x 1 – Correctly differentiates one of the two expressions for time.

	$x^2 = \frac{36}{64}$ $x = \frac{3}{4}$ (also $x > 0$) *Test left and right of 3/4 to show minimum t.p.	
d)	$T = \frac{\sqrt{x^2+1}}{6} + \frac{2-x}{10}$ When $x = \frac{3}{4}, T = \frac{1}{3}$	1 – Correct solution obtained.
30. a)	It is advised to draw a normal distribution graph to illustrate the situation. 595 mL has a z-score of -1. So, $\mathbf{P(595 < X < 600) = 0.34}$ For 602 mL, $z = \frac{602-600}{5} = 0.4$ In the table, $P(z < 0.4) = 0.655$. So, $P(0 < z < 0.4) = 0.655 - 0.5$ $= 0.155$ So, $\boldsymbol{P(600 < X < 602) = 0.155}$ $\therefore \boldsymbol{P(595 < X < 602) = 0.34 + 0.155 = 0.495}$	3 – Correct probabilities obtained. 2 – Correct z-scores determined. 1 – Only 1 z-score determined.
b)	 Using the table, $P(z < 1.6) = 0.945$ So $P(z > 1.6) = 1 - 0.945 = 0.055$ (or 5.5%) By symmetry, x corresponds to a z-score of -1.6 (i.e.: 1.6σ below the mean) $-1.6 = \frac{x-600}{5}$ $x = 592$ mL	2 – Correct solution determined with appropriate mathematical working. 1 – Correct z-score of 1.6 from table obtained.

31. **a)**	$a = 2$ (amplitude) $d = 7$ (middle of sine wave) Period $= \frac{2\pi}{b}$ $4 = \frac{2\pi}{b}$ (highest to lowest point takes 2s, and that is $\frac{1}{2}$ a wavelength. So, 4s for 1 wavelength) $b = \frac{\pi}{2}$	For a total of 4: 1 – Correct amplitude determined. 1 – Correct value of d 2 – Correct value of b with mathematical working.
b)	If weight was initially pushed down, then when $t = 0, x = 5$. $5 = 2\sin\left(\frac{\pi}{2} \cdot 0 + c\right) + 7$ $-1 = \sin(c)$ $c = \frac{3\pi}{2}$ (as $\sin\frac{3\pi}{2} = -1$)	2 – Correct mathematical working and solution provided. 1 – Identifies that $x = 5$ when $t = 0$ and substitutes into formula.
c)	$x = 2\sin\left(\frac{\pi}{2}t + \frac{3\pi}{2}\right) + 7$  	2 – Correct graph provided. 1 – Any one feature of amplitude, period, intercept sketched correctly.